Graph Theoretical Analysis: Current Research, Methodological Issues and Its Applications

严超赣

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Outline

- Principles & Computational Algorithms
- Methodological Issues & Computational Platform
- Applications to Brain Disorders

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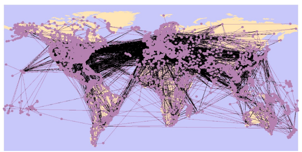
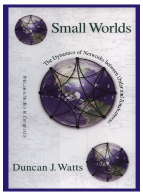
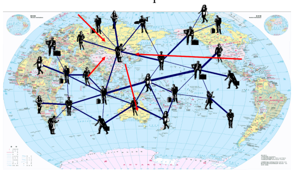
Graph Theoretical Analysis

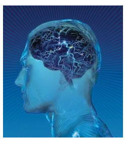


World-wide airport network





Social network



Brain network

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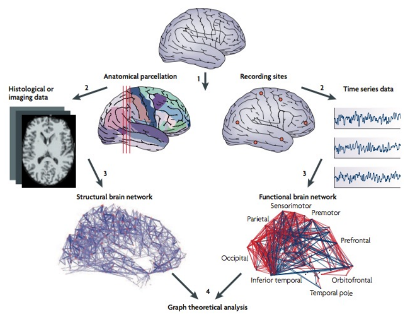
Graph Theoretical Analysis



Bullmore and Sporns, 2009

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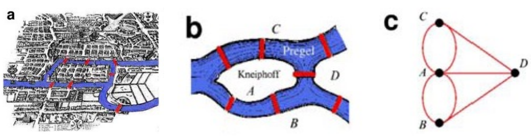
Graph Theoretical Analysis

The Seven Bridges of Königsberg is a historically notable problem in mathematics. Its negative resolution by Leonhard Euler in 1736 laid the foundations of graph theory and prefigured the idea of topology.



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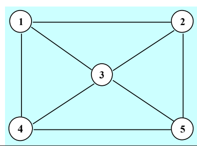
Graph Theoretical Analysis

Graph definition

A graph is composed of a finite non-empty set of vertices and a set of edges between vertices, usually expressed as:

$$G=(V, E)$$

G represents a graph, V is the set of vertices in graph G, and E is the set of edges between vertices in graph G.



$|V(G)|=5, |E(G)|=8$

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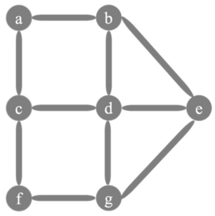
Graph Theoretical Analysis



点	a	b	c	d	e	f	g
a	0	1	1	0	0	0	0
b	1	0	0	1	1	0	0
c	1	0	0	1	0	1	0
d	0	1	1	0	1	0	1
e	0	1	0	1	0	0	1
f	0	0	1	0	0	0	1
g	0	0	0	1	1	1	0

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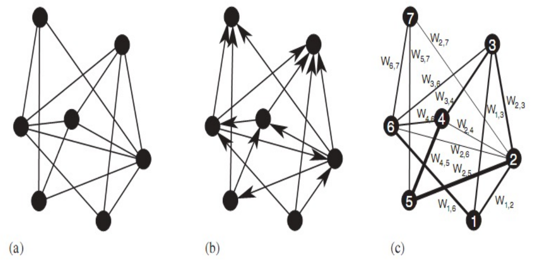
Graph Theoretical Analysis

Type of Graph



a: undirected b: directed c: weighted

[2].S. Boccaletti et al. Physics Reports 424 (2006) 175 –308

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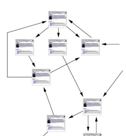
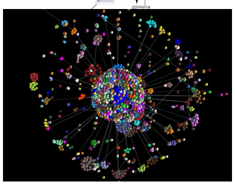
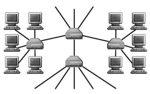
Technology network

WWW

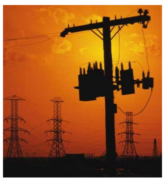




Internet



Power network



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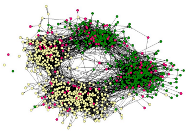
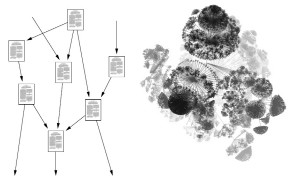
Social network

Friendship network



Citation network



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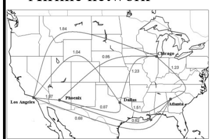
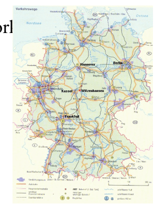
Transportation network

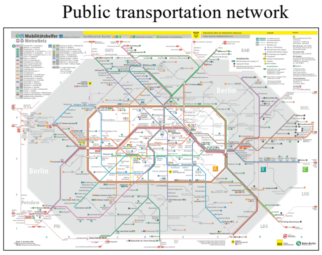
Airline network



Road networl



Public transportation network



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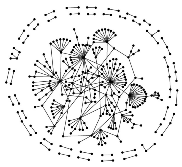
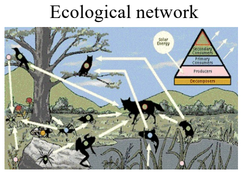
Biological network

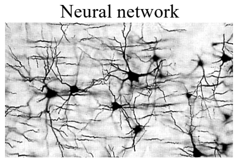
Protein interaction network

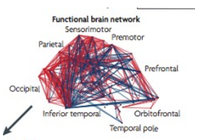


Ecological network



Neural network





Brain network

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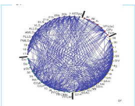
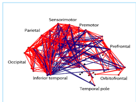
Brain Network: Principles

Tract-tracing



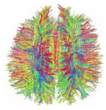
Hilgetag et al., 2000.
Trans R Soc Lond B Biol Sci

Functional MRI



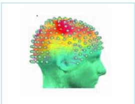
Achard et al., 2006.
J Neurosci

Diffusion MRI



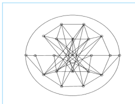
Hagmann et al., 2007.
PLoS ONE

MEG



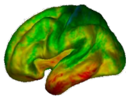
Bassett et al., 2006.
PNAS

EEG



Stam et al., 2007.
Cereb Cortex

Structural MRI



He et al., 2007.
Cereb Cortex

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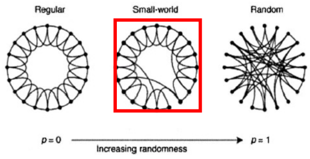
Computational Methodology

Graph theoretical analysis



Watts and Strogatz, 1998. Nature

Node: brain region
Link: connection

Regular: high C_p high L_p

Small-world: high C_p low L_p

Random: low C_p low L_p

Small-world networks contain many local links and a few long-distance links (so-called "shortcuts").

Cp: average clustering of a network
Lp: average shortest path length of a network

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Computational Methodology

Description of the network structure

- Geometric quantities and their distribution

Degree
number of friends

Clustering coefficient
Friends of friends are friends

Shortest path
The path with the least number of edges between two vertices

Betweenness
The number of shortest paths that go through me

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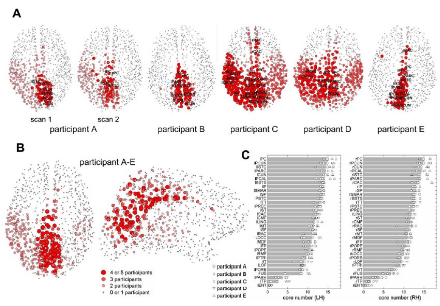
Computational Methodology

- **Brain hubs**



Hagmann et al., 2008. PLoS Biol

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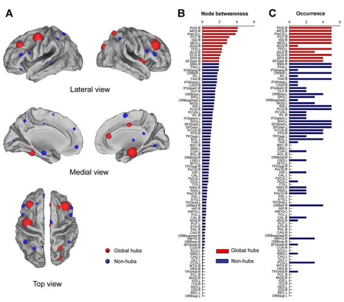
Brain Network: Principles

- **Brain hubs**



He et al., 2009. PLoS ONE

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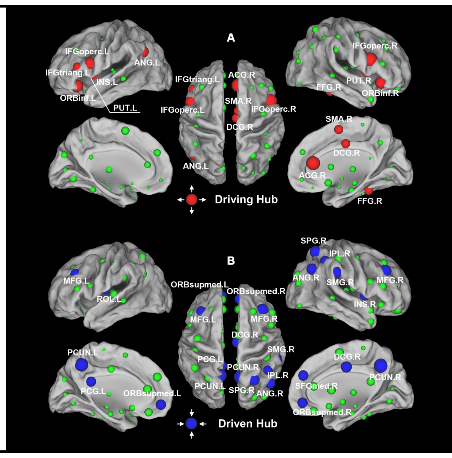

Yan et al., 2011. PLoS ONE

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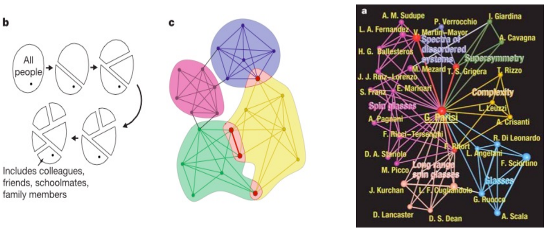
Computational Methodology

Community Structures

The connections within the community are tight, and the connections between the communities are relatively sparse



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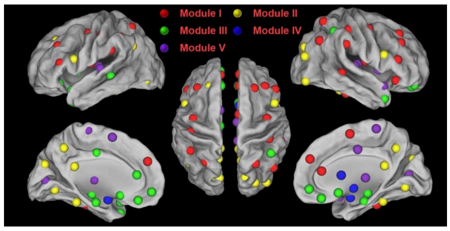
Computational Methodology



Yan et al., 2011. PLoS ONE

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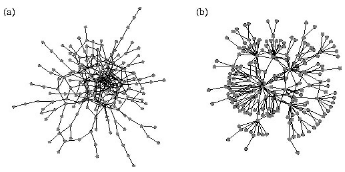
Computational Methodology

Assortativity

- If the vertices with high degree value in the network tend to be interconnected with the vertices of other height values, the network is said to have the same direction matching property; (social network)
- If the vertices with high degree value in the network tend to be interconnected with vertices with low degree value, the network is said to have reverse matching properties; (biological network)



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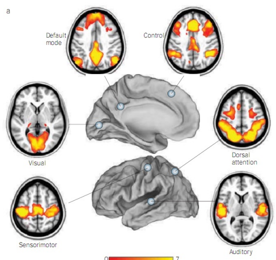
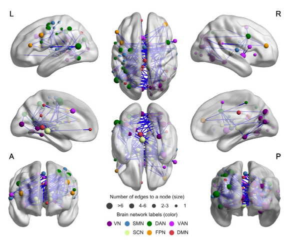
Computational Methodology

- **Correlation**



Zhang and Raichle, 2010. Nat Rev Neurol



Li, et al., 2021. Hum Brain Mapp

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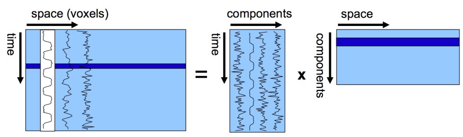
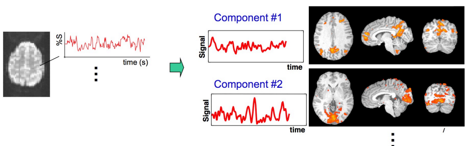
Computational Methodology

- **Independent Component Analysis**





Birn 2015

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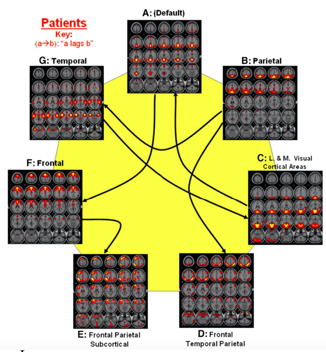
Computational Methodology

- **Functional network connectivity**



Jafri et al., 2008. NeuroImage

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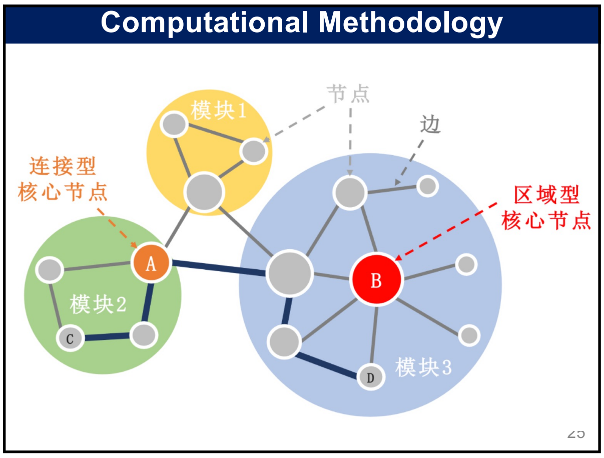

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Computational Methodology

NeuroImage 52 (2010) 1059–1069
Contents lists available at ScienceDirect
NeuroImage
journal homepage: www.elsevier.com/locate/ynimg

Complex network measures of brain connectivity: Uses and interpretations

Mikail Rubinov [a,b,c], Olaf Sporns [d,*]
[a] Black Dog Institute and School of Psychiatry, University of New South Wales, Sydney, Australia
[b] Mental Health Research Division, Queensland Institute of Medical Research, Brisbane, Australia
[c] CSIRO Information and Communication Technologies Centre, Sydney, Australia
[d] Department of Psychological and Brain Sciences, Indiana University, Bloomington, IN 47405, USA

Rubinov and Sporns, 2010. NeuroImage

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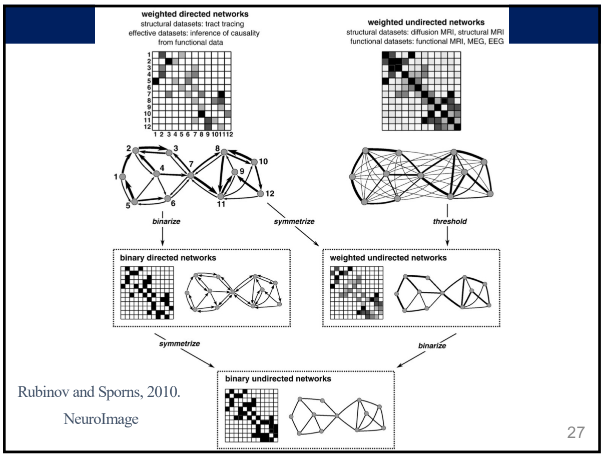

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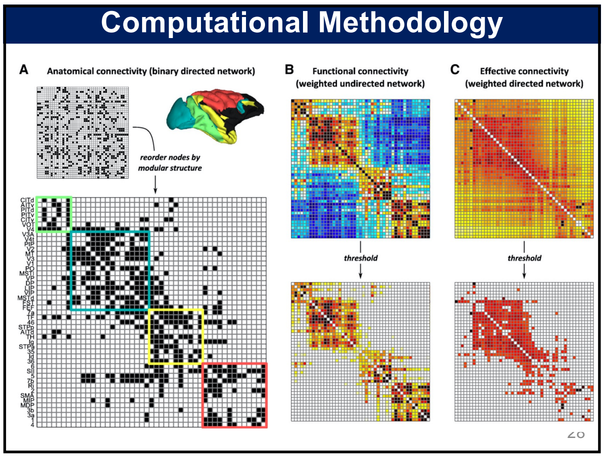

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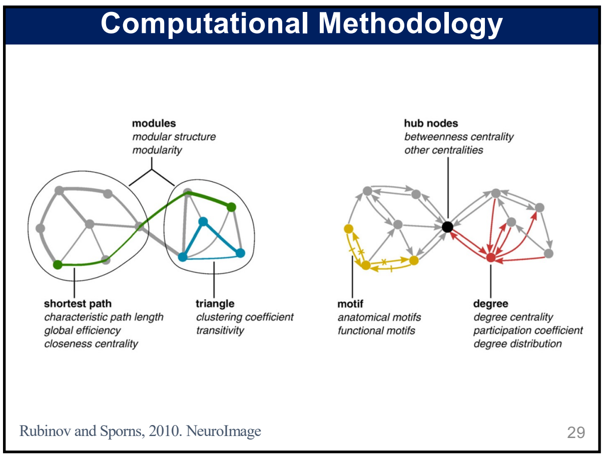

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Computational Methodology

Table A1
Mathematical definitions of complex network measures (see supplementary information for a self-contained version of this table).

Measure	Binary and undirected definitions	Weighted and directed definitions
Basic concepts and measures		
Basic concepts and notation	N is the set of all nodes in the network, and n is the number of nodes. L is the set of all links in the network, and l is number of links. (i, j) is a link between nodes i and j, $(i, j \in N)$. a_{ij} is the connection status between i and j: $a_{ij} = 1$ when link (i, j) exists (when i and j are neighbors); $a_{ij} = 0$ otherwise ($a_{ii} = 0$ for all i). We compute the number of links as $l = \sum_{i,j \in N} a_{ij}$ (to avoid ambiguity with directed links we count each undirected link twice, as a_{ij} and as a_{ji}).	Links (i, j) are associated with connection weights w_{ij}. Henceforth, we assume that weights are normalized, such that $0 \le w_{ij} \le 1$ for all i and j. l^w is the sum of all weights in the network, computed as $l^w = \sum_{i,j \in N} w_{ij}$. Directed links (i, j) are ordered from i to j. Consequently, in directed networks a_{ij} does not necessarily equal a_{ji}.
Degree: number of links connected to a node	Degree of a node i, $k_i = \sum_{j \in N} a_{ij}$.	Weighted degree of i, $k_i^w = \sum_{j \in N} w_{ij}$. (Directed) out-degree of i, $k_i^{out} = \sum_{j \in N} a_{ij}$. (Directed) in-degree of i, $k_i^{in} = \sum_{j \in N} a_{ji}$.
Shortest path length: a basis for measuring integration	Shortest path length (distance), between nodes i and j, $d_{ij} = \sum_{a_{uv} \in g_{i \leftrightarrow j}} a_{uv}$, where $g_{i \leftrightarrow j}$ is the shortest path (geodesic) between i and j. Note that $d_{ij} = \infty$ for all disconnected pairs i, j.	Shortest weighted path length between i and j, $d_{ij}^w = \sum_{a_{uv} \in g_{i \leftrightarrow j}^w} f(w_{uv})$, where f is a map (e.g., an inverse) from weight to length and $g_{i \leftrightarrow j}^w$ is the shortest weighted path between i and j. Shortest directed path length from i to j, $d_{ij}^{\rightarrow} = \sum_{a_{ij} \in g_{i \rightarrow j}} a_{ij}$, where $g_{i \rightarrow j}$ is the directed shortest path from i to j.
Number of triangles: a basis for measuring segregation	Number of triangles around a node i, $t_i = \frac{1}{2} \sum_{j,h \in N} a_{ij} a_{ih} a_{jh}$.	(Weighted) geometric mean of triangles around i, $t_i^w = \frac{1}{2} \sum_{j,h \in N} (w_{ij} w_{ih} w_{jh})^{1/3}$. Number of directed triangles around i, $t_i^{\rightarrow} = \frac{1}{2} \sum_{j,h \in N} (a_{ij} + a_{ji})(a_{ih} + a_{hi})(a_{jh} + a_{hj})$.
Measures of integration		
Characteristic path length	Characteristic path length of the network (e.g., Watts and Strogatz, 1998), $L = \frac{1}{n} \sum_{i \in N} L_i = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \ne i} d_{ij}}{n-1}$, where L_i is the average distance between node i and all other nodes.	Weighted characteristic path length, $L^w = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \ne i} d_{ij}^w}{n-1}$. Directed characteristic path length, $L^{\rightarrow} = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \ne i} d_{ij}^{\rightarrow}}{n-1}$.

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Computational Methodology

frontiers in
HUMAN NEUROSCIENCE

ORIGINAL RESEARCH ARTICLE
published: 26 December 2013
doi: 10.3389/fnhum.2013.00910

Addressing head motion dependencies for small-world topologies in functional connectomics

Chao-Gan Yan[1,2,3], R. Cameron Craddock[1,2], Yong He[4,5] and Michael P. Milham[1,2]**

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[2] Center for the Developing Brain, Child Mind Institute, New York, NY, USA
[3] The Phyllis Green and Randolph Cowen Institute for Pediatric Neuroscience, New York University Child Study Center, New York, NY, USA
[4] State Key Laboratory of Cognitive Neuroscience and Learning & IDG/McGovern Institute for Brain Research, Beijing Normal University, Beijing, China
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Yan et al., 2013. Front Hum Neurosci.

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Computational Methodology

Table 1 | Topological properties of brain graphs examined in the current study.

Topological properties	Descriptions
GLOBAL TOPOLOGICAL PROPERTIES	
Local efficiency	The average efficiency of information transfer over a node's direct neighbors
Global efficiency	The efficiency of information transfer through the entire graph
Clustering coefficient	The average inter-connectedness of a node's direct neighbors
Characteristic shortest path length	The average shortest path length between any pairs of nodes
Normalized clustering coefficient	The clustering coefficient compared to matched random networks
Normalized characteristic shortest path length	The characteristic shortest path length compared to matched random networks
Small-worldness	The normalized clustering coefficient divided by the normalized characteristic shortest path length, which reflect the balance of global efficiency and local efficiency
Assortativity	The tendency of nodes to link with those nodes with similar number of edges
Modularity	The extent to which a graph can be segregated into densely intraconnected but sparsely interconnected modules

Yan et al., 2013.
Front Hum Neurosci.

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Computational Methodology

REGIONAL TOPOLOGICAL PROPERTIES	
Degree centrality	The number (or sum of weights) of connections connected directly to a node
Nodal efficiency	The efficiency of information transfer over a node's direct neighbors
Nodal clustering coefficient	The inter-connectedness of a node's direct neighbors
Subgraph centrality	The participation of a node in all subgraphs comprised in a graph
Betweenness centrality	The influences of a node over information flow between other nodes
Eigenvector centrality	A self-referential measure of centrality – nodes have high eigenvector centrality if they connect to other nodes that have high eigenvector centrality

Yan et al., 2013.
Front Hum Neurosci.

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Outline

- **Resting-State fMRI: Principles**
- **Data Analysis: Computational Algorithms**
- **Data Analysis: Methodological Issues**
- **Data Analysis: Computational Platform**
- **Applications to Brain Disorders**

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Methodological Issues

NeuroImage 211 (2020) 116604

Contents lists available at ScienceDirect

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ELSEVIER

Optimising network modelling methods for fMRI

Usama Pervaiz[a,*], Diego Vidaurre[b,c], Mark W. Woolrich[b], Stephen M. Smith[a]

[a] Oxford Centre for Functional MRI of the Brain (FMRIB), Wellcome Centre for Integrative Neuroimaging, Nuffield Department of Clinical Neurosciences, University of Oxford, Oxford, OX3 9DU, United Kingdom
[b] Oxford Centre for Human Brain Activity (OHBA), Wellcome Centre for Integrative Neuroimaging, Department of Psychiatry, University of Oxford, Oxford, OX3 7JX, United Kingdom
[c] Department of Clinical Medicine, Aarhus University, Denmark

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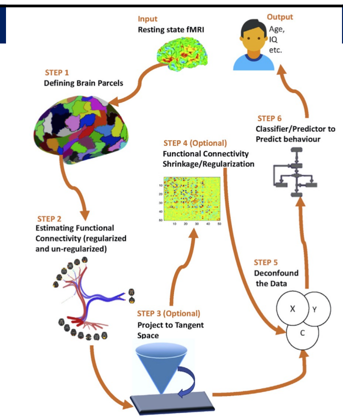

Fig. 1. Flow chart summarizing the six major steps of the methodology framework for rfMRI analysis.

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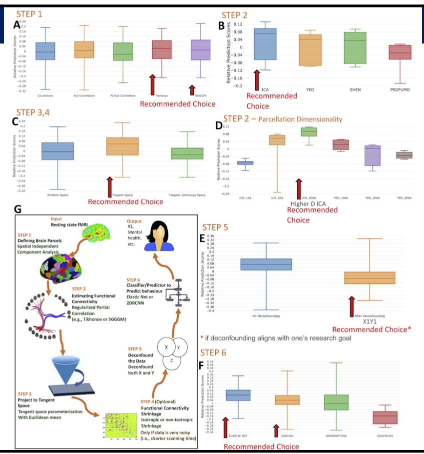

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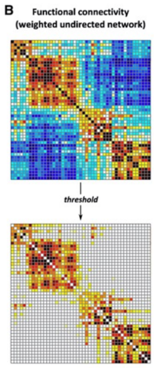
Methodological Issues

Thresholding



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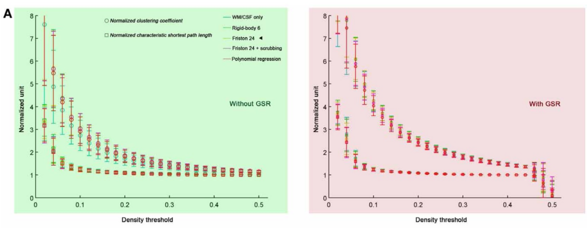
Methodological Issues



Yan et al., 2013. Front Hum Neurosci.

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Methodological Issues

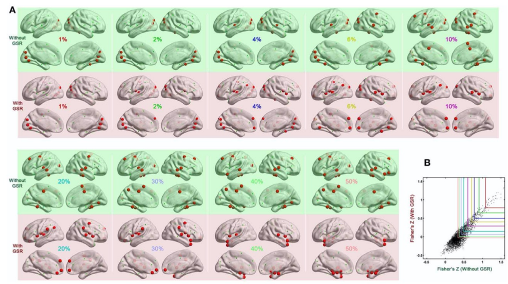

FIGURE 3 | Impact of density on hub distribution. (A) Hub distribution across various densities either without GSR (green shaded) or with GSR (pink shaded) derived from the data corrected with Friston 24 model. With stringent density thresholds, the hub distributions are similar between data with and without GSR. When the density increases, the discrepancy of hub distribution between with and without GSR becomes dominant. **(B)** Scatter plot of Fisher's Z averaged across participants. Most of the top connections can be identified either with or without GSR. However, when the percentage increases, a large portion of connections can be only identified by one procedure but not the other.

Yan et al., 2013. Front Hum Neurosci.

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Methodological Issues

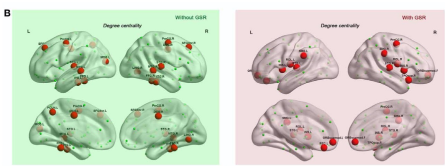
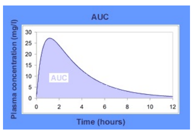


FIGURE 2 | Small-world properties (A) and hub distributions (B) under different head motion correction strategies. Five preprocessing strategies were evaluated in combination without GSR (green shaded) and with GSR (pink shaded). "◄" in panel **(A)** indicates that the hub distribution demonstrated in panel **(B)** is derived from Friston 24 model. The hub distribution was demonstrated with the area under the curve (AUC) of degree centrality integrated within density range of 0.06–0.44, while regions with AUC > mean + SD are considered as hubs. For the abbreviations of the regions, see **Table A1**. Surface maps were created with BrainNet Viewer (Xia et al., 2013 www.nitrc.org/projects/bnv/).



Yan et al., 2013. Front Hum Neurosci.

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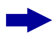
Outline

- Resting-State fMRI: Principles
- Data Analysis: Computational Algorithms
- Data Analysis: Methodological Issues
-  Data Analysis: Computational Platform
- Applications to Brain Disorders

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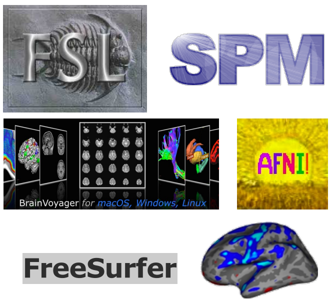
Traditional fMRI Preprocessing Toolbox



- Numerous steps and configurations
- High learning curve
- Big data era of neuroimaging calls for new pipelines

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Computational sharing platform for fMRI

- ➢ Incorporating DPARSF

Prior work, cited 2704 times

- ➢ Adapting methodological updates

Head motion (cited 1159 times)

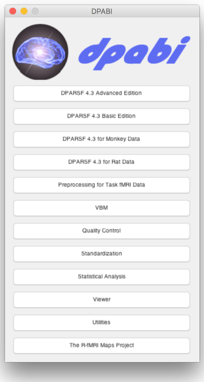
Standardization (cited 340 times)

Multiple comparison correction (cited 176 times)

- ➢ Standardized preprocessing pipeline
- ➢ Statistical toolbox
- ➢ Platform for data sharing

Yan et al., 2016. Neuroinformatics

Corresponding author



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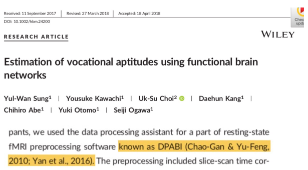
Peer Evaluation

Cited by 1532 times, ESI Top 1‰ top cited paper and hot paper


RESEARCH ARTICLE

WILEY

Estimation of vocational aptitudes using functional brain networks

Yul-Wan Sung[1] | Yousuke Kawachi[1] | Uk-Su Choi[2] | Daehun Kang[1] | Chihiro Abe[1] | Yuki Otomo[1] | Seiji Ogawa[1]

pants, we used the data processing assistant for a part of resting-state fMRI preprocessing software known as DPABI (Chao-Gan & Yu-Feng, 2010; Yan et al., 2016). The preprocessing included slice-scan time cor-

Seiji Ogawa

Inventor of fMRI BOLD

45

45

DPARSF

Data Organization

ProcessingDemoData.zip

FunRaw

Sub_001

Sub_002

Sub_003

T1Raw

Sub_001

Sub_002

Sub_003

Functional DICOM data

Structural DICOM data

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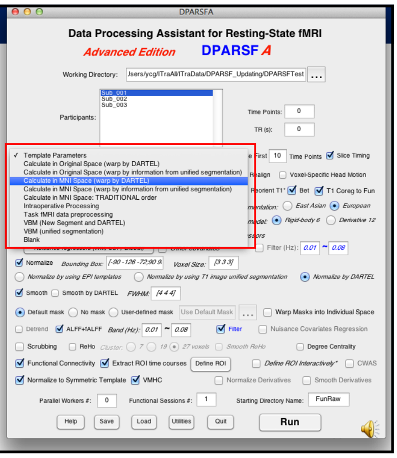

Resting State fMRI Data Processing

Template Parameters

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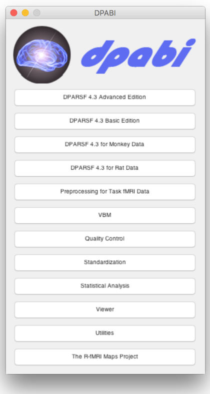
DPABI

Yan et al., 2016. Neuroinformatics

ESI Top 0.1% Highly Cited Paper



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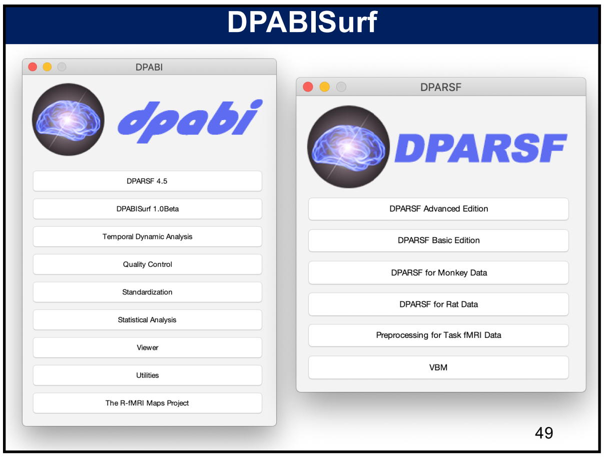

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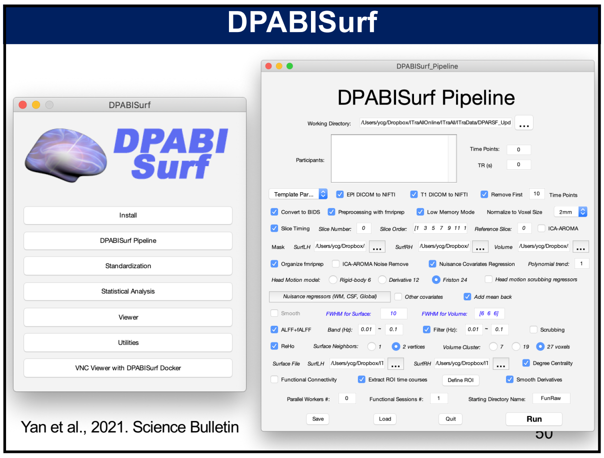

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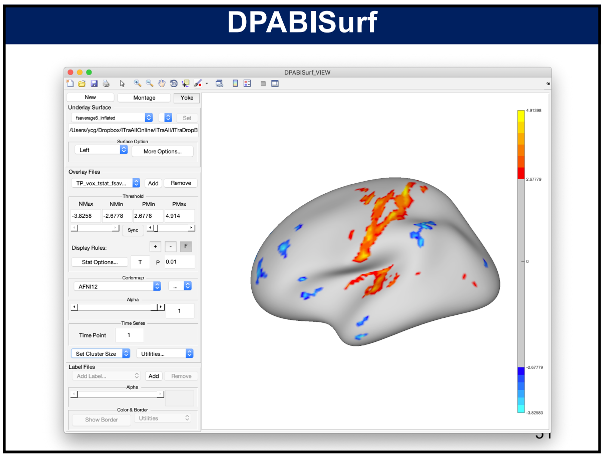

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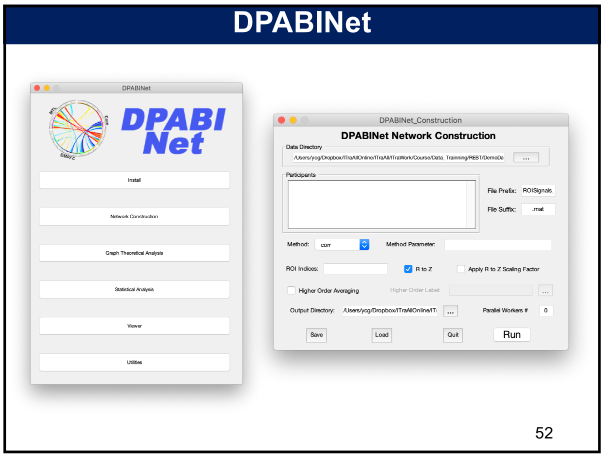

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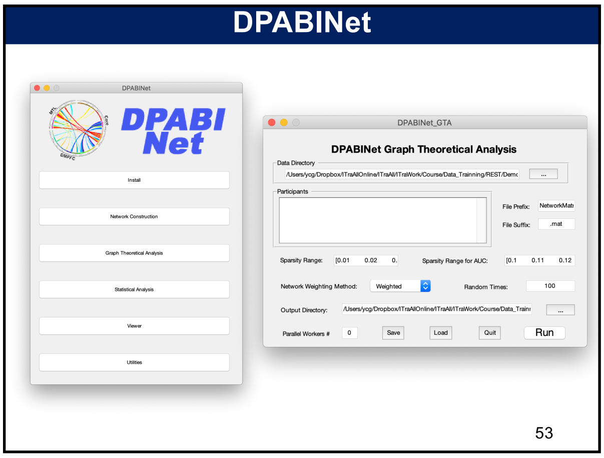

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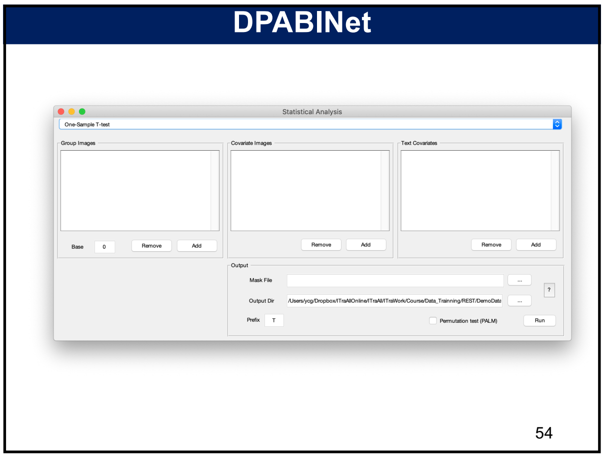

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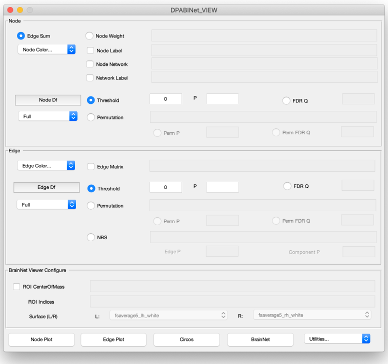
DPABINet



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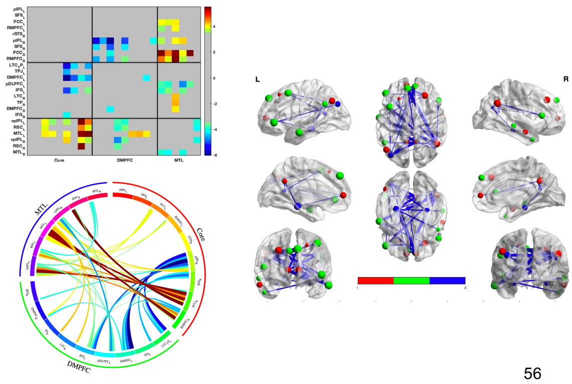

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Future Directions

- R-fMRI methodology
- Mechanism of R-fMRI: electrophysiology/fMRI recording
- Modulation and intervention: medication and brain stimulation
- Application to brain disorders

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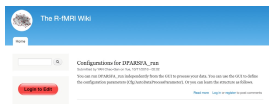
Further Help

http://rfmri.org/Course



http://rfmri.org/wiki

The R-fMRI Journal Club

Official Account: RFMRILab

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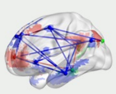
DPABI/DPABISurf/DPARSF特训营

第九届DPABI/DPABISurf/DPARSF
脑影像基础特训营（云端）通知
2021.3.27～3.29



第一届DPABISurf/DPABINet
脑网络进阶特训营（北京现场）通知
2021.4.24～4.26

定期举办，请关注http://rfmri.org

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DPABISurf皮层数据预处理业务

特训营学员：99元/例次

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The R-fMRI Lab

WeChat Official Account: RFMRILab

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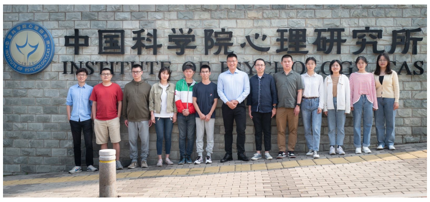
Acknowledgments



Chinese Academy of Sciences
Xi-Nian Zuo
Wei-Wen Wang
Fei Luo

Hangzhou Normal University
Yu-Feng Zang

NYU Child Study Center
F. Xavier Castellanos

Peking University Sixth Hospital
Tian-Mei Si
Jing Liu

Funding
- National Natural Science Foundation of China
- National Key R&D Program of China
- Chinese Academy of Sciences

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Thanks for your attention!

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